GLIMPSES on the MICRO BLACK HOLE

PLANCK PHASE

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- MICRO B.H. ARE SUPPOSED TO BE CREATED AT LHC IN THE NEXT FUTURE

- Much work has been done to predict and understand the decay behaviour and signatures of such events (GIDDINGS, MYERS, EMPARAN, CAVAGLIA', HARMS, ...)

- 3 PHASES: ROTATING PHASE (a rotating m.b.h. is created, it looses energy and angular momentum until it settles down to the Schwarzschild phase) (SUPER-RADIANCE + HAW. RAD

  SCHWARZSCHILD PHASE (m.b.h. looses mass/energy through the HAWKING EFFECT: SEMICLASSICAL APPROXIMATION) (HAWKING EFF. is computed via the usual HEISENBERG UNCERTAINTY P.)

  PLANCK (or quantum gravity) PHASE (STILL POORLY UNDERSTOOD: WE DONT HAVE A COMPLETE THEORY OF QUANTUM GRAVITY)

- HOWEVER: A good approxim. of the QG regime can be obtained by using a G.U.P. (i.e. deformed commutators) to compute TEMPERATURE, MASS THRESHOLDS, b.h. LIFETIMES

The calculation of QFT on Curved Spacetime (with deformed commutators) can be safely obtained by employing the (Generalised) Uncertainty Principle and some basic physics [SCARDIGLI '95, ADLER CHEN SANTIAGO 2001, CAVAGLIA' DAS MAARTENS 2003]

- EXAMPLE: Computation of the Hawking temperature from H.U.P.

$$\Delta x \, \Delta E \simeq \hbar c/2 \; ; \qquad \Delta x \simeq 2R_s = \frac{4GM}{c^2} \; ; \qquad \Delta E \simeq 3KT \text{ (photons!)}$$

Δx: UNCERTAINTY IN THE POSITION OF THE HAWKING QUANTUM.

ΔE: THERMAL ENERGY OF THE EMITTED QUANTUM ($\gamma, g, \ldots$)

$$\Rightarrow T = \frac{\hbar c^3}{24 G K M} \qquad \left[\text{QFT FORMULA: } T = \frac{\hbar c^3}{8\pi G K M} \right]$$

$8\pi \simeq 24$

- Example: Hawking temperature from the GUP in 4 DIM

$$\Delta x \gtrsim \frac{\hbar c}{2\Delta E} + 2\ell_p^2 \frac{\Delta E}{\hbar c} \Rightarrow \frac{\Delta E}{\hbar c} = \frac{\Delta x}{2\ell_p^2}\left(1-\sqrt{1-\frac{4\ell_p^2}{\Delta x^2}}\right) \Rightarrow$$

$$\Rightarrow T_{GUP} = \frac{Mc^2}{4\pi}\left(1-\sqrt{1-\frac{m_p^2}{M^2}}\right) \Rightarrow M \gtrsim m_p$$: PREDICTION OF REMNANTS

- 2 KINDS OF G.U.P. IN 4+n DIM

- STRINGY INSPIRED G.U.P.: $\Delta x \gtrsim \frac{\hbar c}{2\Delta E} + 2\beta \ell_{4n}^2 \frac{\Delta E}{\hbar c} = \frac{\hbar c}{2\Delta E} + \beta \ell_{4n} \frac{\Delta E}{\mathcal{E}_{4n}}$

WHERE: ℓ_{4n} = 4(+n) dim. PLANCK LENGTH
$\mathcal{E}_{4n}$ = 4(+n) dim. PLANCK ENERGY
β = deformation parameter [of $O(1)$, CAN BE ESTIMATED]

THIS GUP COMES FROM STRING THEORY AND SCATTERING EXPERIMENTS (GEDANKEN)

- MICRO BLACK HOLE INSPIRED G.U.P.

$$\Delta x \gtrsim \frac{\hbar c}{2\Delta E} + \beta R_{4n}(\Delta E) = \frac{\hbar c}{2\Delta E} + \beta (\omega_n)^{\frac{1}{n+1}} \ell_{4n} \left(\frac{\Delta E}{\mathcal{E}_{4n}}\right)^{\frac{1}{n+1}}$$

WHERE: $R_{4n}(\Delta E)$ IS THE SCHWARZSCHILD RADIUS ASSOCIATED WITH ΔE

$\left[\omega_n = \frac{8\pi}{(N-1)\Omega_{N-1}}\right.$, Ω_{N-1} = AREA OF THE UNIT S^{N-1} SPHERE, $\left.\omega_0 = 1\right]$

THIS GUP CAN BE OBTAINED BY SCATTERING GEDANKEN EXPERIMENTS INVOLVING THE FORMATIONS OF MICRO BLACK HOLES. ESSENTIALLY:

$$\Delta x \gtrsim \begin{cases} \frac{\hbar c}{2\Delta E} & \Delta E < \mathcal{E}_{4n} \\ R_{4n}(\Delta E) & \Delta E > \mathcal{E}_{4n} \end{cases}$$

ALTERNATIVELY: VERY H.E. PHOTONS

$$\lambda = \frac{\hbar c}{\mathcal{E}} \longrightarrow \lambda = \frac{\hbar c}{\mathcal{E}} + \beta R_{4n}(\mathcal{E})$$

REMARKABLY, IN 4 DIM THE 2 PRINCIPLES COINCIDE!
(So that the deformation parameter β can be chosen the same for both)

- FOR THIS GUP, β CAN BE FIXED:

IF ΔE LARGE $\Rightarrow \Delta x \gtrsim \beta R_{4n}(\Delta E)$
BUT SHOULD ALSO BE: $\Delta x \gtrsim 2 R_{4n}$
$\Rightarrow \beta \simeq 2$

• From the GUPs to the MASS-TEMPERATURE Relation

UNCERTAINTY IN THE POSITION OF A QUANTUM OF HAWKING RADIATION $\Big\}$ $\Delta x = 2R_{4n} = 2\,\ell_{4n}(\omega_n m)^{\frac{1}{n+1}}$

$m = M/M_{4n}$ $\qquad$ $N = 3+n =$ # SPACELIKE DIMENSIONS

⊙ STRINGY GUP: $$2(\omega_n m)^{\frac{1}{n+1}} = \frac{\mathcal{E}_{4n}}{\Delta E} + \beta\,\frac{\Delta E}{\mathcal{E}_{4n}}$$

⊙ MBH GUP: $$2(\omega_n m)^{\frac{1}{n+1}} = \frac{\mathcal{E}_{4n}}{\Delta E} + \beta\left(\omega_n\,\frac{\Delta E}{\mathcal{E}_{4n}}\right)^{\frac{1}{n+1}}$$

• RELATION BETWEEN ΔE AND THE TEMPERATURE

- THE MINIMUM B.H. MASSES (MASS THRESHOLDS) ARE INDIPENDENT FROM THE ANALYTIC FORM OF $\Delta E(T)$ (provided ΔE is a monotonically increasing function of T)
- SUPPOSING A LINEAR RELATION $\Delta E = \alpha(n)T$ and requiring the matching to the HAWKING LIMIT (LOW ENERGY LIMIT, $\beta \to 0$) WE FIND $\Delta E(T) = \dfrac{\pi K_B T}{n+1}$ AND THE MASS-TEMPERATURE RELATIONS ARE:

(ST) $$2(\omega_n m)^{\frac{1}{n+1}} = \frac{n+1}{2\pi\theta} + \beta\,\frac{2\pi\theta}{n+1}$$

(MBH) $$2(\omega_n m)^{\frac{1}{n+1}} = \frac{n+1}{2\pi\theta} + \beta\left(\frac{2\pi\omega_n\theta}{n+1}\right)^{\frac{1}{n+1}}$$

WHERE $\theta = \dfrac{T}{T_{4n}}$ and $\mathcal{E}_{4n} = \frac{1}{2}K_B T_{4n}$

• MASS THRESHOLDS

- WE CAN STUDY VARIOUS LIMITS for the function $\frac{\Delta E}{\varepsilon_{4n}}$: $\begin{cases} \Gamma_g \ll \Gamma_\gamma \text{ (BRANE)} \\ \Gamma_g \gg \Gamma_\gamma \text{ (BULK)} \end{cases}$

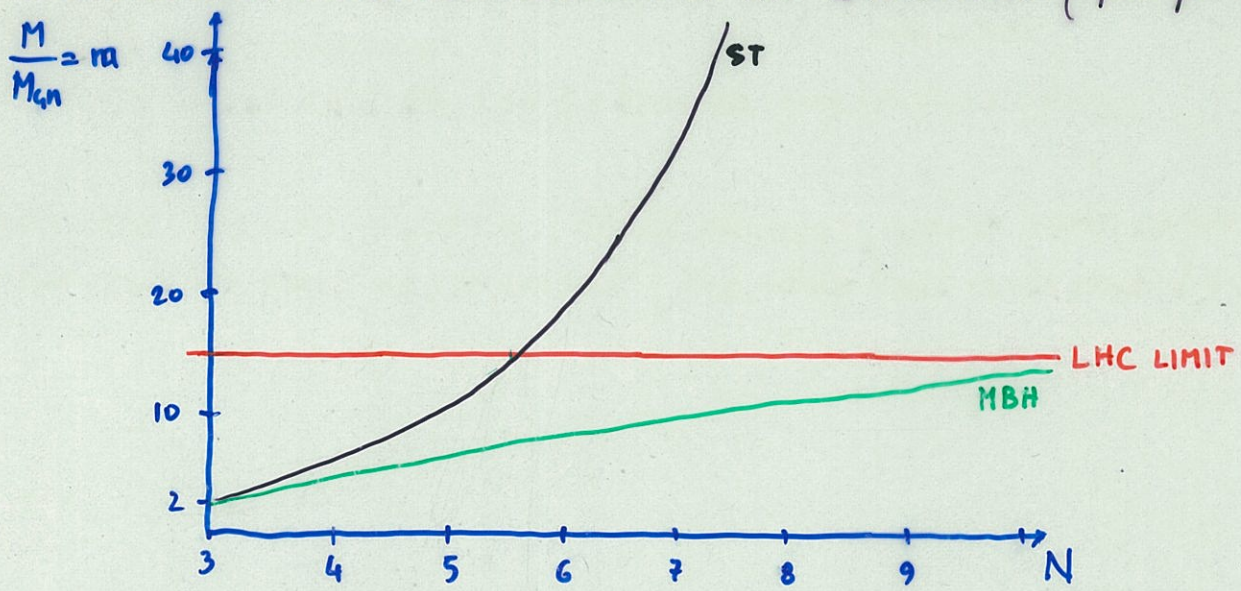
- HOWEVER, IT CAN BE SHOWN THAT THE MINIMUM B.H. MASSES (MASS THRESHOLDS) ARE INDEPENDENT FROM THE TWO LIMITS $\Gamma_g \ll \Gamma_\gamma$, $\Gamma_g \gg \Gamma_\gamma$

- FOR THE MINIMUM ALLOWED MASSES WE GET :

$$m_{MIN}^{ST} = \frac{\beta^{\frac{1}{2}(N-2)}}{\omega_n} \qquad \text{STRINGY GUP}$$

$$m_{MIN}^{MBH} = \left[\frac{N-1}{2(N-2)^{\frac{N-2}{N-1}}}\right]^{(N-2)} \cdot (\omega_n)^{\frac{1}{1-N}} \cdot \beta^{\frac{(N-2)^2}{N-1}} \qquad \text{MICRO B.H. GUP}$$

- IT IS INTERESTING TO COMPARE THE PLOTS OF THE 2 FUNCTIONS (for $\beta = 2$)



- THE MBH GUP ALLOWS THE CREATION OF MICRO BH BELOW THE LHC LIMIT FOR ANY NUMBER OF (EXTRA) DIMENSIONS

- ON THE CONTRARY : THE STRINGY GUP PREDICTS MASSES FOR THE MICRO B.H. ALREADY OUTSIDE THE ENERGY RANGE OF LHC FOR $N=6$ ($n=3$ EXTRA DIM)

• MBH GUP

$$\begin{cases} -\dfrac{d(2y)}{d\tau} = \Gamma_\gamma \tilde{G}_n (2y)^{5-N} \Lambda^{4(N-2)} + \Gamma_g \tilde{H}_n (2y)^2 \Lambda^{(N-2)(N+1)} \\ 2y = \dfrac{\omega_n}{\Lambda^{N-2}} + \beta\Lambda \quad \text{WITH} \quad \Lambda = (\omega_n \lambda \theta)^{\frac{1}{N-2}} \end{cases}$$

BH LIFETIME: $$\tau^{MBH} = \int_{\Lambda_{MAX}}^{\Lambda_0(m_0)} \mathcal{J}(\Lambda)\, d\Lambda$$

WHERE $\Lambda_{MAX} = \left[\dfrac{(N-2)\,\omega_n}{\beta}\right]^{\frac{1}{N-1}}$; Λ_0 = SMALLEST POSITIVE ROOT OF

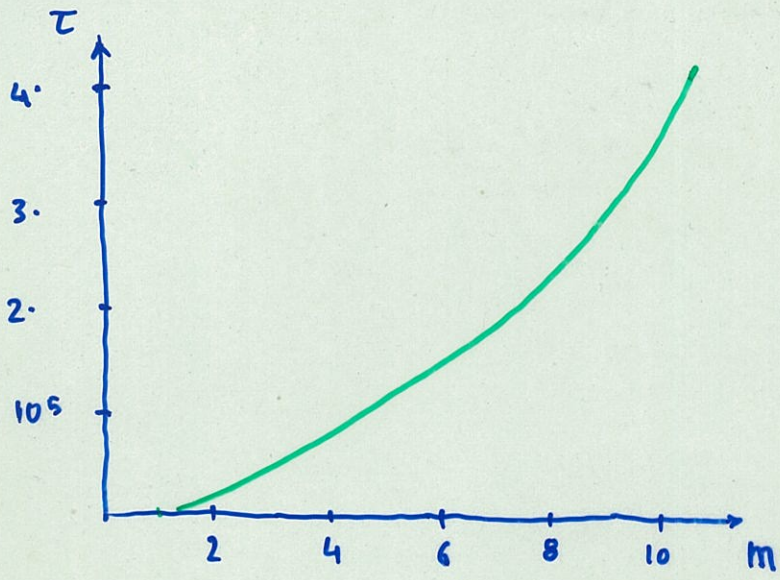
$$\beta\Lambda_0^{N-1} - 2(\omega_n m_0)^{\frac{1}{N-2}} \Lambda_0^{N-2} + \omega_n = 0$$

AND

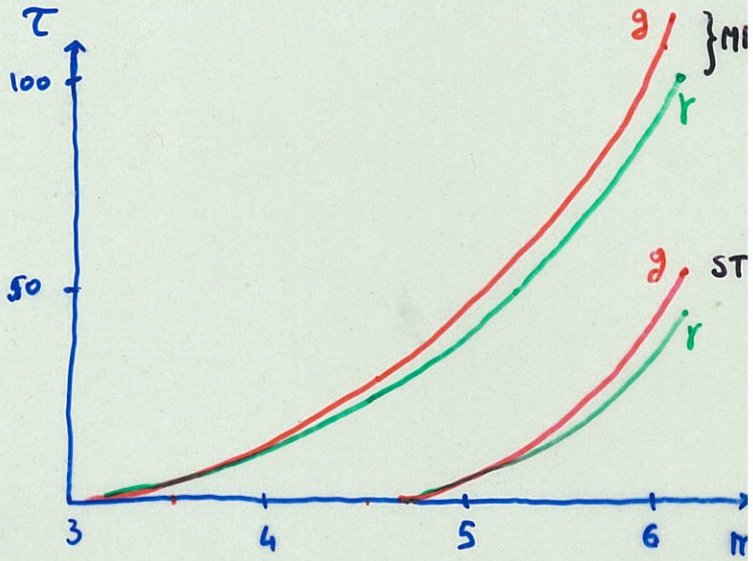
$$\mathcal{J}(\Lambda) = \frac{(\beta\Lambda^{N-1} - (N-2)\omega_n)}{\left[\Gamma_\gamma \tilde{G}_n (\beta\Lambda^{N-1} + \omega_n)^{3-N} + \Gamma_g \tilde{H}_n\right](\beta\Lambda^{N-1} + \omega_n)^2 \Lambda^{(N-1)^2}}$$

• SOME PLOTS OF THE FUNCTIONS : $\tau^{ST}(m_0, N, \Gamma_\gamma, \Gamma_g, \beta)$

$\tau^{MBH}(m_0, N, \Gamma_\gamma, \Gamma_g, \beta)$



N = 3



N = 4

• MICRO BLACK HOLE LIFETIMES

• EMISSION RATE EQUATION

$$-\frac{dE}{dt} = \underbrace{3\,\Gamma_\gamma\,\sigma_3\,\Omega_2\,R_{4n}^2\,T^4}_{\substack{\text{PHOTONS}\\ \text{(ON THE BRANE)}}} + \underbrace{N\,\Gamma_g\,\sigma_N\,\Omega_{N-1}\,R_{4n}^{N-1}\,T^{N+1}}_{\substack{\text{GRAVITONS}\\ \text{(BRANE + BULK)}}}$$

Γ_γ, Γ_g = GREY BODY FACTORS

• EMISSION RATE EQUATIONS IN PLANCK UNITS

$$-\frac{dm}{d\tau} = 2\,\Gamma_\gamma\,\Omega_2\,\frac{\Gamma(4)\,\zeta(4)}{\pi^4}\,(\omega_n m)^{\frac{2}{N-2}}\,\theta^4 +$$

$$+ (N+1)(N-2)\,\Gamma_g\,\frac{\Omega_{N-1}^2}{(2\pi)^N}\,\Gamma(N+1)\,\zeta(N+1)\,(\omega_n m)^{\frac{N-1}{N-2}}\,\theta^{N+1}$$

• ANALYTICAL INTEGRATION

- SETTING $y = (\omega_n m)^{\frac{1}{N-2}}$, $\lambda = \frac{2\pi}{N-2}$ WE HAVE

⊙ STRINGY GUP

SEPARABLE EQUATION

$$\begin{cases} -\dfrac{d(2y)}{d\tau} = \Gamma_\gamma\,G_n\,(2y)^{5-N}\,\bar\theta^4 + \Gamma_g\,H_n\,(2y)^2\,\bar\theta^{N+1} \\[2ex] 2y = \dfrac{1}{\bar\theta} + \beta\,\bar\theta \qquad \text{WITH} \qquad \bar\theta = \lambda\theta \end{cases}$$

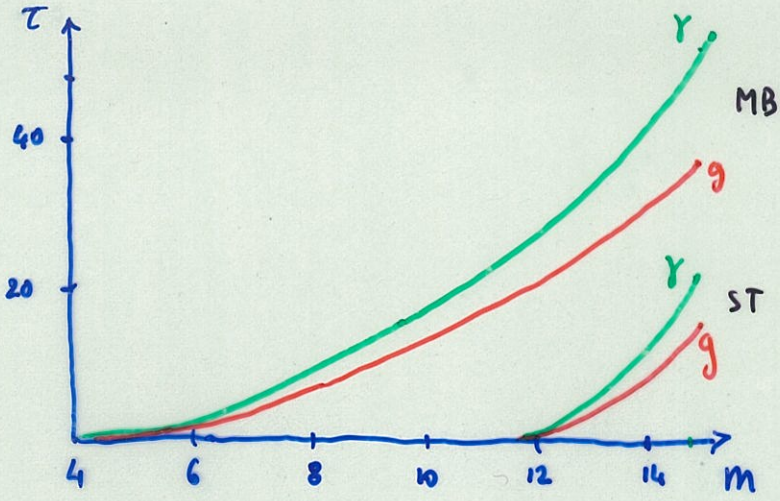
LIFETIME : $$\tau^{ST} = \int_{\bar\theta_{MAX}}^{\bar\theta_0(m_0)} \mathcal{G}_N(\bar\theta)\,d\bar\theta$$

WHERE $\bar\theta_{MAX} = \frac{1}{\sqrt{\beta}}$; $\bar\theta_0$: SMALLEST POSITIVE ROOT OF

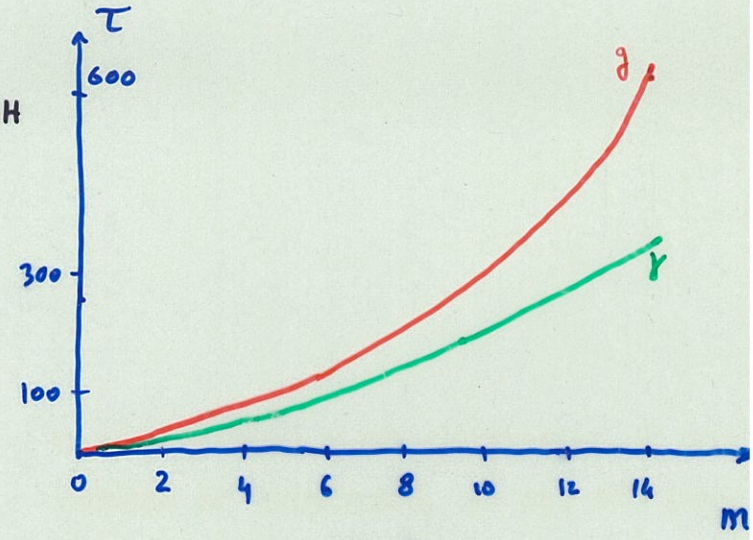
$$\beta\,\bar\theta_0^2 - 2(\omega_n m_0)^{\frac{1}{N-2}}\,\bar\theta_0 + 1 = 0$$

AND

$$\mathcal{G}_N(\bar\theta) = \frac{\beta\bar\theta^2 - 1}{\left[\Gamma_\gamma\,G_n(\beta\bar\theta^2+1)^{3-N} + \Gamma_g\,H_n\right](\beta\bar\theta^2+1)^2\,\bar\theta^{N+1}}$$



N = 5



N = 5

HAWKING LIMIT, $\beta \to 0$

THE GREEN LINES REPRESENT EMISSIONS DUE TO PHOTONS ONLY $(\Gamma_\gamma = 1,\ \Gamma_g = 0)$

THE RED LINES ARE THE EMISSIONS DUE TO GRAVITONS ONLY $(\Gamma_\gamma = 0,\ \Gamma_g = 1)$

• CONCLUSIONS

The MBH GUP brings to predictions sensibly different from those given by the ST GUP. Essentially:

- MASS THRESHOLDS LOWER (MICRO B.H. SHOULD BE DETECTABLE IN ANY NUMBER OF EXTRA DIM., AT THE DESIGNED ENERGY FOR LHC)

- MICRO B.H. LIFETIMES LONGER IN MBH GUP THAN IN ST GUP.

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