Black rings in more than five dimensions

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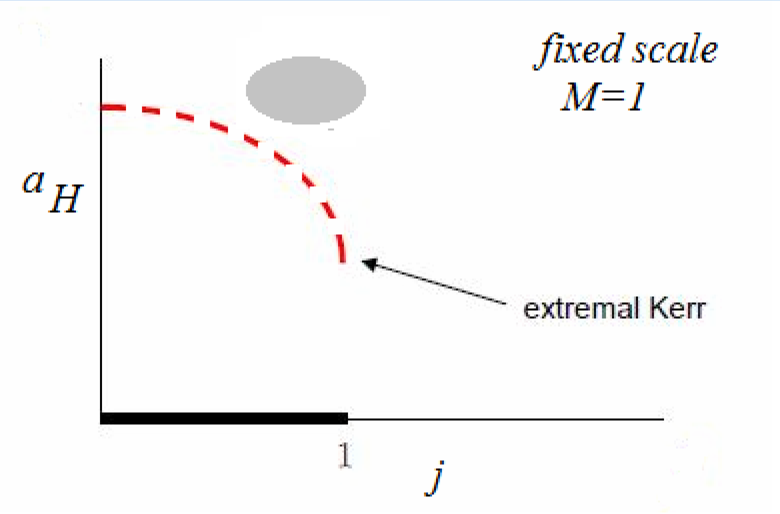
4d *vs.* higher d Black Holes

(asymptotically Minkowski solutions only!)



GR in four dimensions

- the topology of the horizon: a sphere S^2
- “no hair” theorems
- Kerr black hole (uniqueness)

why study BHs in *d>4* dimensions?

no obvious contact with reality…

However:

- dimension - a parameter of GR:
  interesting mathematical problem
- string theory/large extra dimensions
- hints for AdS solutions -> duality

d>4: the novel feature –
non-uniqueness of black objects

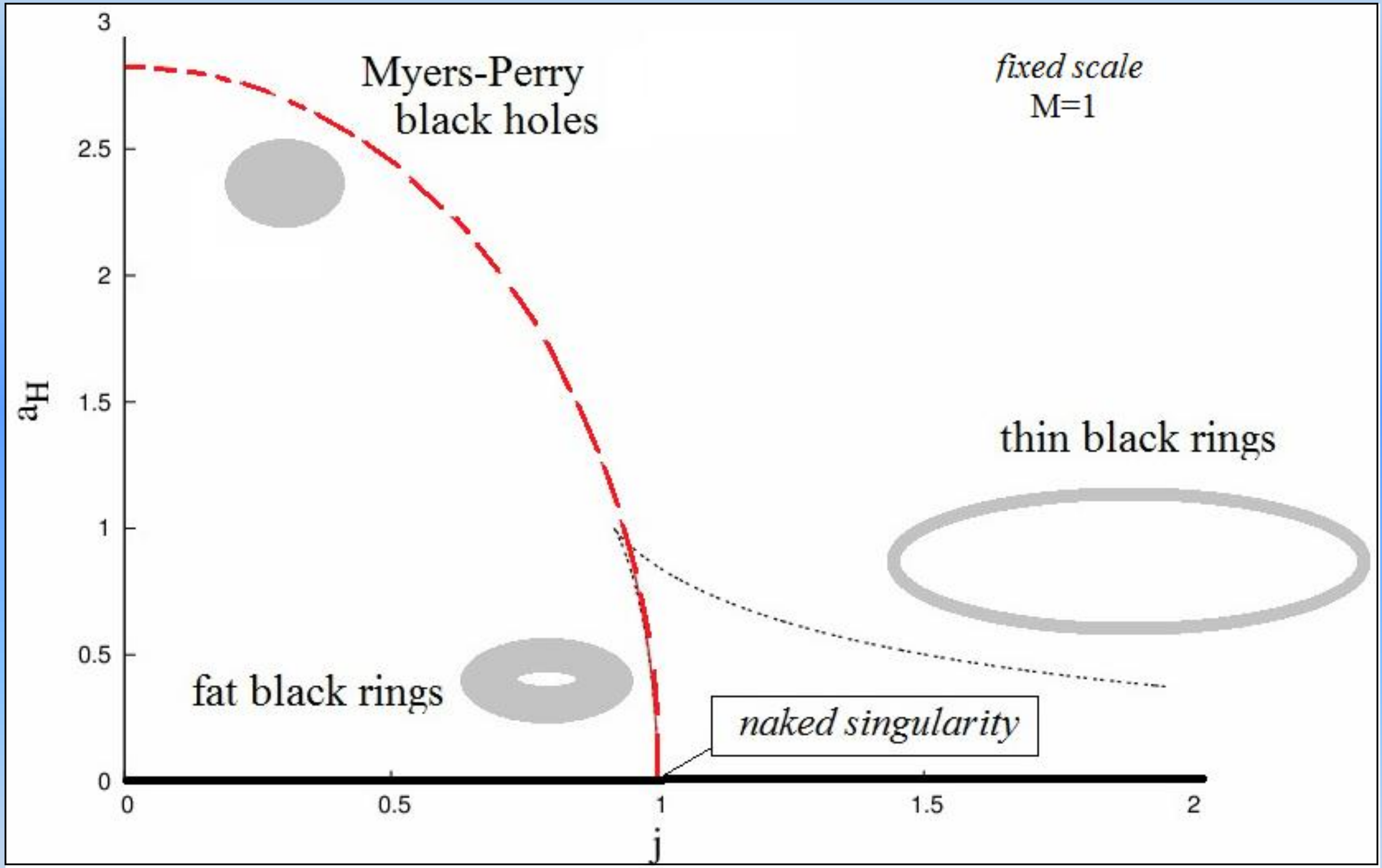
- richer rotation dynamics
- extended black objects:
  (*black strings/branes*)

⇒ **many new types of solutions**

Last decade: huge progress for d=5

One-black hole phases in five dimensions:

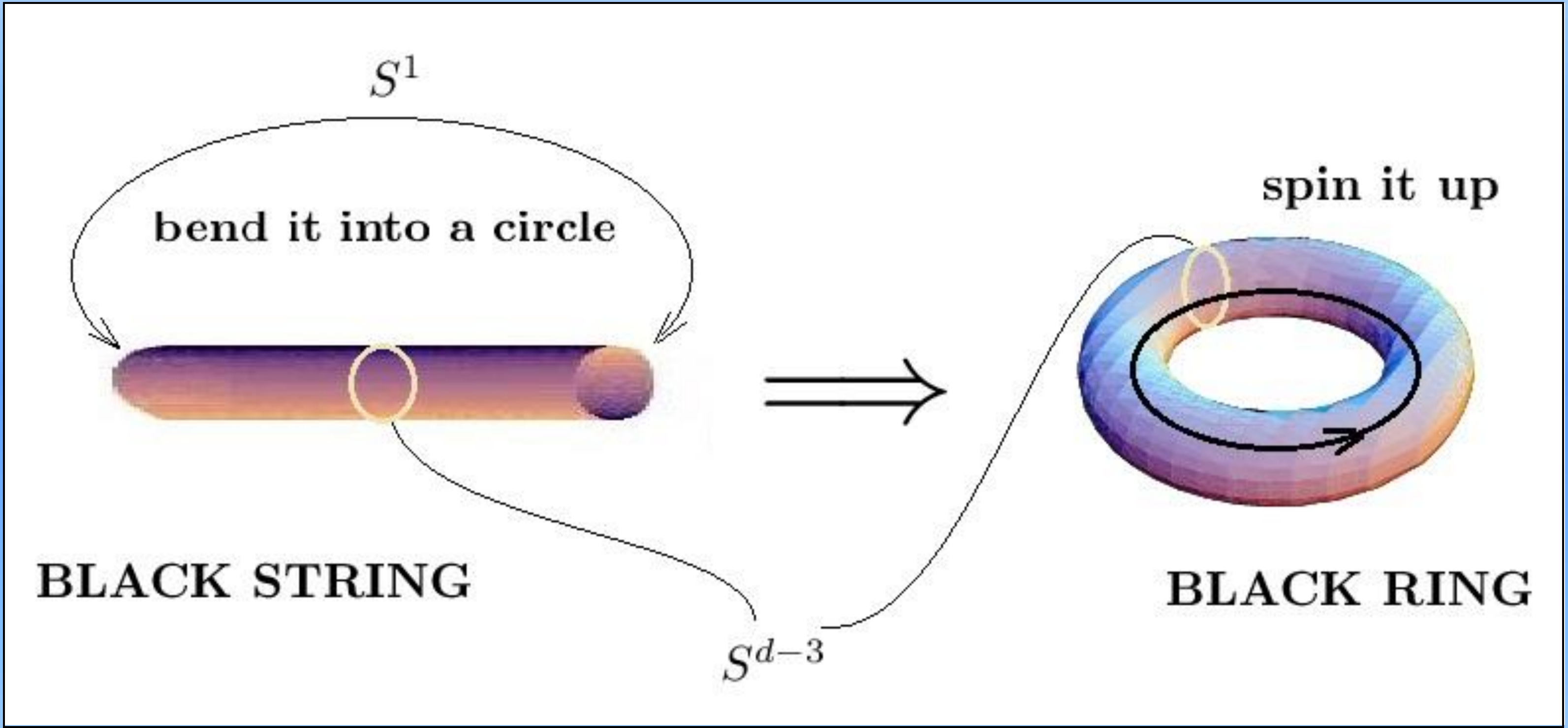
(rotation in a single plane)



- ***the black ring exact solution (Emparan-Reall 2002)***
- ***three different black holes with the same value of (M,J)***
- ***non uniqueness!***

However, the black rings should exist also in more than five dimensions

Schwarzschild black hole in (d-1)-dimensions: black string in d-dimensions



<u>five dimensions</u>

•there's an explicit realisation, and fairly simple *(Emparan-Reall 2002)*

<u>d>5</u>

•approximate solution based on this heuristic construction (large ring radius)

(Emparan, Harmark, Niarchos, Obers, Rodriguez, 2007)

Black rings in more than five dimensions

(arXiv: 1205.5437)

Crucial ingredient: **A new coordinate system**

$\boldsymbol{E_D}$:

$$ds^2 = d\rho^2 + \rho^2(d\Theta^2 + \cos^2\Theta d\Omega^2_{D-3} + \sin^2\Theta d\psi^2)$$

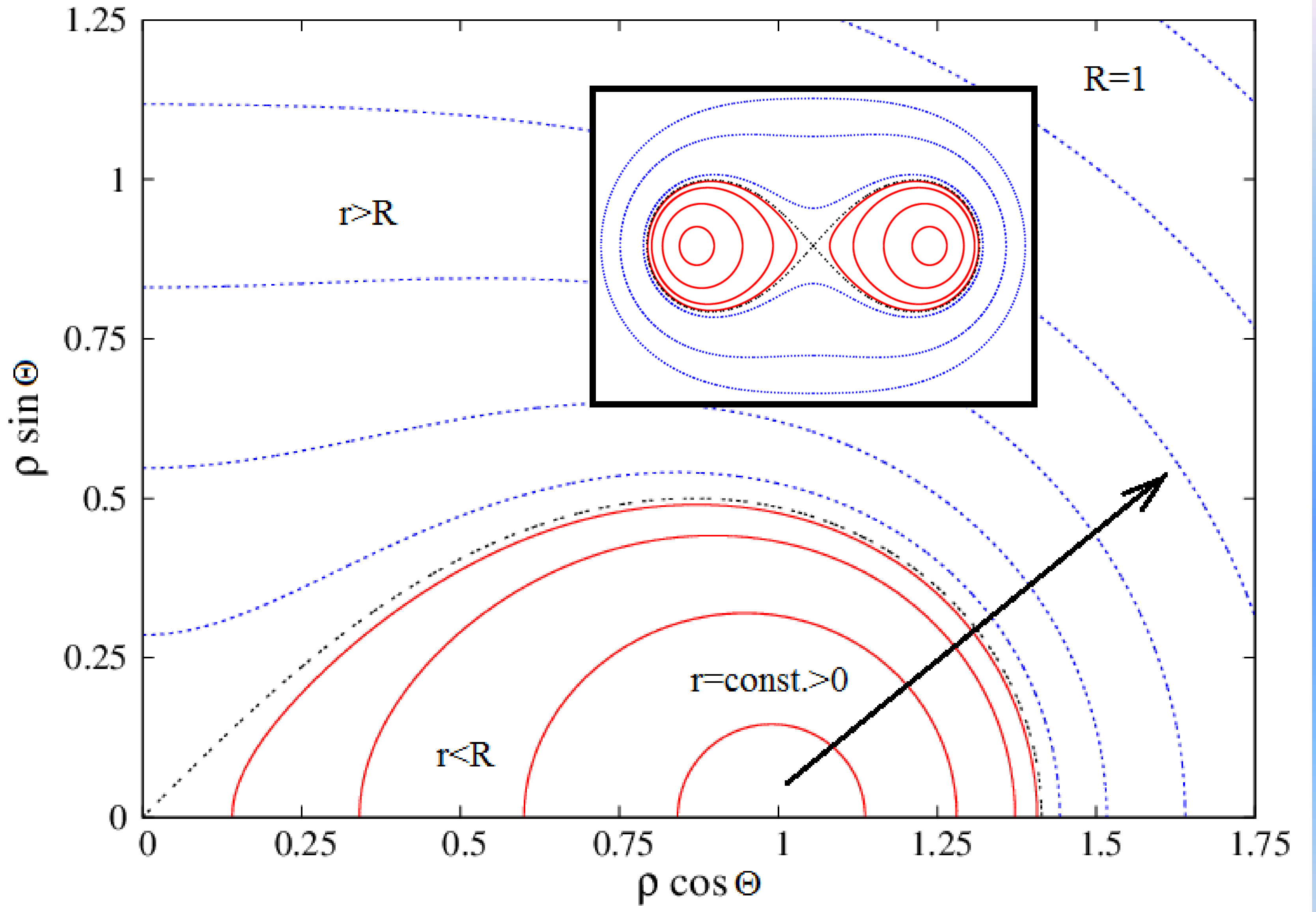
$$\rho = r\sqrt{U}, \quad \tan\Theta = \left(\tfrac{r^2+\rho^2+R^2}{r^2+\rho^2-R^2}\right)\tan\theta,$$

$$ds^2_D = V_1(dr^2 + r^2 d\theta^2) + V_2 d\Omega^2_{D-3} + V_3 d\psi^2,$$

$$V_1 = \frac{1}{U}, \quad V_2 = r^2\left(\cos^2\theta - \frac{1}{2}(1 + \frac{R^2}{r^2} - U)\right), V_3 = r^2\left(\sin^2\theta - \frac{1}{2}(1 - \frac{R^2}{r^2} - U)\right)$$

$$\text{with } U = \sqrt{1 + \frac{R^4}{r^4} - \frac{2R^2}{r^2}\cos 2\theta}.$$

convenient for numerics: rectangular shape $0 \le r < \infty,\ 0 \le \theta \le \pi/2$



for $0 < r < R$, a surface of constant r has ring-like topology $S^{D-2} \times S^1$

how to construct the black rings?

general metric ansatz:

$$ds^2 = f_1(r,\theta)(dr^2 + r^2 d\theta^2) + f_2(r,\theta)d\Omega^2_{d-4} + f_3(r,\theta)(d\psi - w(r,\theta)dt)^2 - f_0(r,\theta)dt^2.$$

Q: ***is this ansatz too restrictive?***

A: ***no, many solutions can be written in this form***

Example: *d=5 Emparan-Reall balanced black ring*

$$w = -4\sqrt{2}\frac{r^3 r_H^2 R(r^2 + r_H^2)^2(r_H^2 + R^2)\sqrt{r_H^4 + R^4}}{(r^2 - r_H^2)^2(R^2 - r_H^2)}\frac{S_2 S_3}{Q},$$

with

$$S_2 = -(r^4 + r_H^4)R^2 + r^2(r_H^2 + R^2)^2 - 2r^2 r_H^2 R^2 \cos 2\theta + 2R^2 r^2 U,$$

$$S_3 = (r^4 - 4r^2 r_H^2 + r_H^4)(r_H^4 + R^4) + 4r^2 r_H^4 R^2 \cos 2\theta - 4r^2 r_H^2 R^2 U,$$

$$U = \frac{1}{2r^2 R^2}\sqrt{(r^4 + R^4 - 2r^2 R^2 \cos 2\theta)(r_H^8 + r^4 R^4 - 2r^2 r_H^4 R^2 \cos 2\theta)}$$

etc…

the equations we solve:

$R_{ik}=0$

$$\nabla^2 f_0 + \frac{(d-4)}{2f_2}\nabla f_0\cdot\nabla f_2 - \frac{1}{2f_0}(\nabla f_0)^2 + \frac{1}{2f_3}\nabla f_0\cdot\nabla f_3 - f_3(\nabla w)^2 = 0,$$

$$\nabla^2 f_1 - \frac{1}{f_1}(\nabla f_1)^2 - \frac{1}{4}(d-4)(d-5)\frac{f_1}{f_2^2}(\nabla f_2)^2 - \frac{(d-4)}{2}\frac{f_1}{f_0 f_2}\nabla f_0\cdot\nabla f_2 - \frac{(d-4)}{2}\frac{f_1}{f_3 f_2}\nabla f_2\cdot\nabla f_3$$
$$- \frac{1}{2}\frac{f_1}{f_0 f_3}\nabla f_0\cdot\nabla f_3 - \frac{1}{2}\frac{f_1 f_3}{f_0}(\nabla w)^2 + (d-4)(d-5)\frac{f_1^2}{f_2} = 0,$$

$$\nabla^2 f_2 + \frac{1}{2}(d-6)(\nabla f_2)^2 + \frac{1}{2f_0}\nabla f_0\cdot\nabla f_2 + \frac{1}{2f_3}\nabla f_2\cdot\nabla f_3 - 2(d-5)f_1 = 0,$$

$$\nabla^2 f_3 - \frac{1}{2f_3}(\nabla f_3)^2 + \frac{(d-4)}{2f_2}\nabla f_2\cdot\nabla f_3 + \frac{1}{2f_0}\nabla f_0\cdot\nabla f_3 + \frac{f_3^2}{f_0}(\nabla w)^2 = 0,$$

$$\nabla^2 w - \frac{1}{2f_0}\nabla f_0\cdot\nabla w + \frac{(d-4)}{2f_2}\nabla f_2\cdot\nabla w + \frac{3}{2f_3}\nabla f_3\cdot\nabla w = 0.$$

with

$$(\nabla U)\cdot(\nabla V) = \partial_r U\partial_r V + \tfrac{1}{r^2}\partial_\theta U\partial_\theta V,$$

$$\nabla^2 U = \partial_r^2 U + \tfrac{1}{r^2}\partial_\theta^2 U + \tfrac{1}{r}\partial_r U$$

$\boxed{(f_i, w)}$ – encode all properties of the solutions

$r \geq r_H$ only

The metric of a spatial cross-section of the horizon

$$d\sigma^2 = f_1(r_H,\theta) r_H^2 d\theta^2 + f_2(r_H,\theta) d\Omega_{d-4}^2 + f_3(r_H,\theta) d\psi^2.$$

$$A_H = 2\pi r_H V_{d-4} \int_0^{\pi/2} d\theta \sqrt{f_1 f_2^{d-4} f_3}\Big|_{r=r_H}$$

event horizon area

$$T_H = \frac{1}{2\pi} \lim_{r \to r_H} \frac{1}{(r-r_H)} \sqrt{\frac{f_0}{f_1}},$$

temperature

$$\Omega_H = w|_{r=r_H},$$

event horizon velocity

+

mass and angular momentum: ***computed from asymptotics***

$$g_{tt} = -f_0 = -1 + \frac{c_t}{r^{d-3}} + \ldots, \quad g_{\psi t} = -f_3 w = \sin^2\theta \frac{c_\psi}{r^{d-3}} + \ldots$$

$$M = \frac{(d-2)V_{d-2}}{16\pi G} c_t, \quad J = \frac{V_{d-2}}{8\pi G} c_\psi.$$

Numerical procedure

solve numerically the Einstein equations (= a set of PDEs in two variables)

the input:
even horizon radius r_H
event horizon velocity Ω_H
radius of the ring R

numerics: two methods

- finite difference
- spectral solver

the output:
the functions (f_i ,w)
+ the derivatives

test the numerics
- recover known solutions *(e.g. the d=5 black ring)*
- verify the Smarr law

$$(d-3)M = (d-2)(T_H \frac{A_H}{4G} + \Omega_H J).$$

TEST OF NUMERICS:

our results vs. the Emparan-Reall exact solution (d=5)

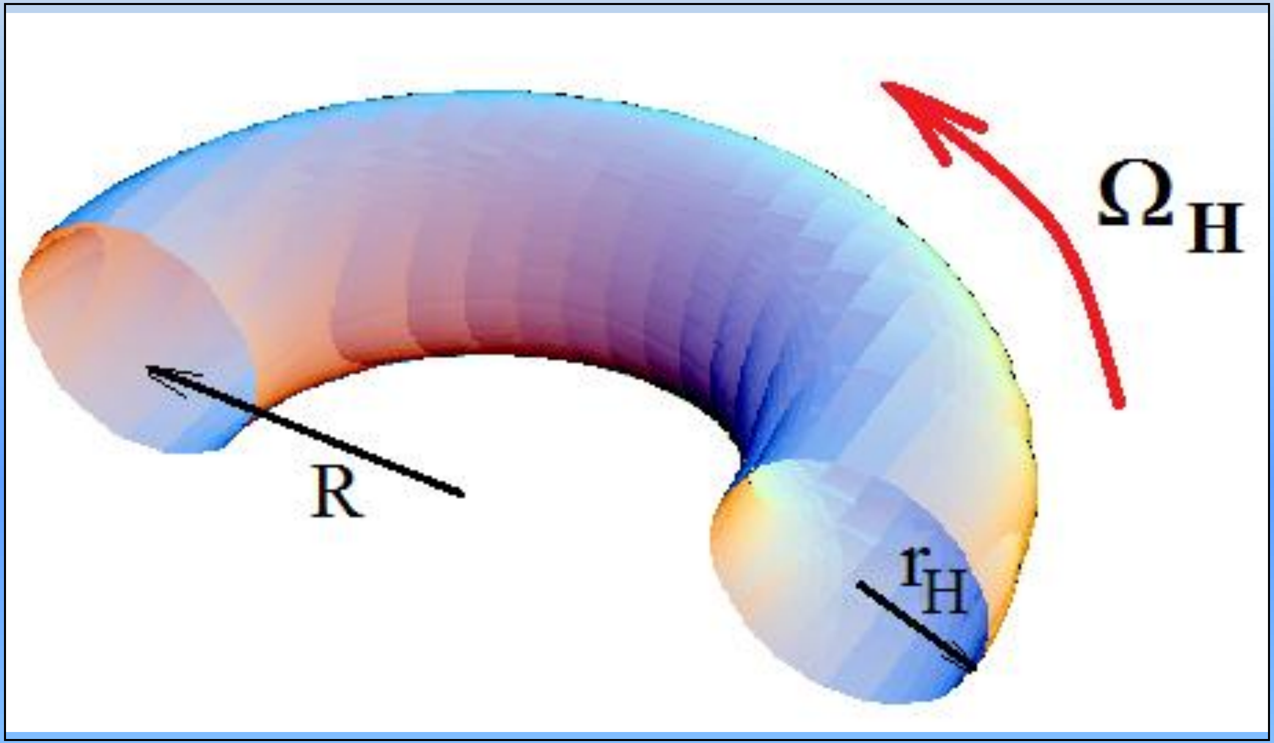
R	Ω_H	$M(num)$	$M(ex)$	$\|J(num)\|$	$\|J(ex)\|$	$A_H(num)$	$A_H(ex)$
1.61803	0.182574	24.0003	24.0000	109.547	109.545	773.616	773.605
1.93186	0.204124	16.0000	16.0000	58.7880	58.7878	446.647	446.645
2.18890	0.207020	13.3332	13.3333	45.0836	45.0843	332.911	332.909
2.41421	0.204124	12.0001	12.0000	39.1922	39.1918	273.514	273.518
2.80588	0.193649	10.6671	10.6667	34.4289	34.4265	210.552	210.563
3.14626	0.182574	9.99982	10.0000	32.8624	32.8634	176.553	176.555
3.45197	0.172516	9.59981	9.60000	32.4596	32.4607	154.723	154.726
3.99215	0.155902	9.14274	9.14286	32.9869	32.9877	127.614	127.616
4.46653	0.143019	8.88879	8.88889	34.1828	34.1834	110.970	110.973

- accuracy in the numerics: around 10^{-5}

thus similar techniques for d>5 will result in reliable numerical results

In numerics: **three input parameters:** $R,\ r_H,\ \Omega_H$

$$0 \le r_H \le R < \infty$$

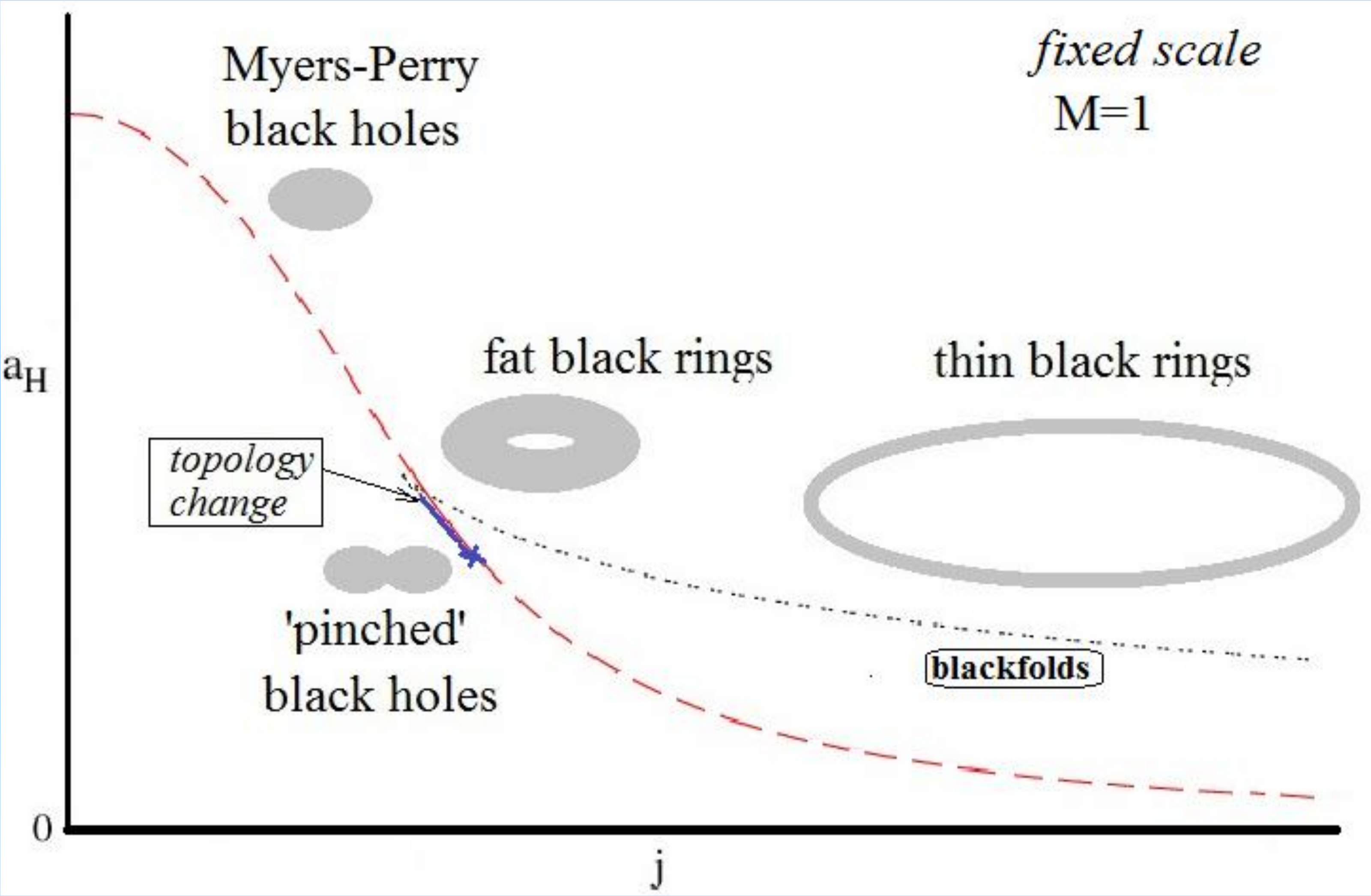


$T_H,\ A_H,\ M,\ J$ **-computed from numerical output**

In practice:

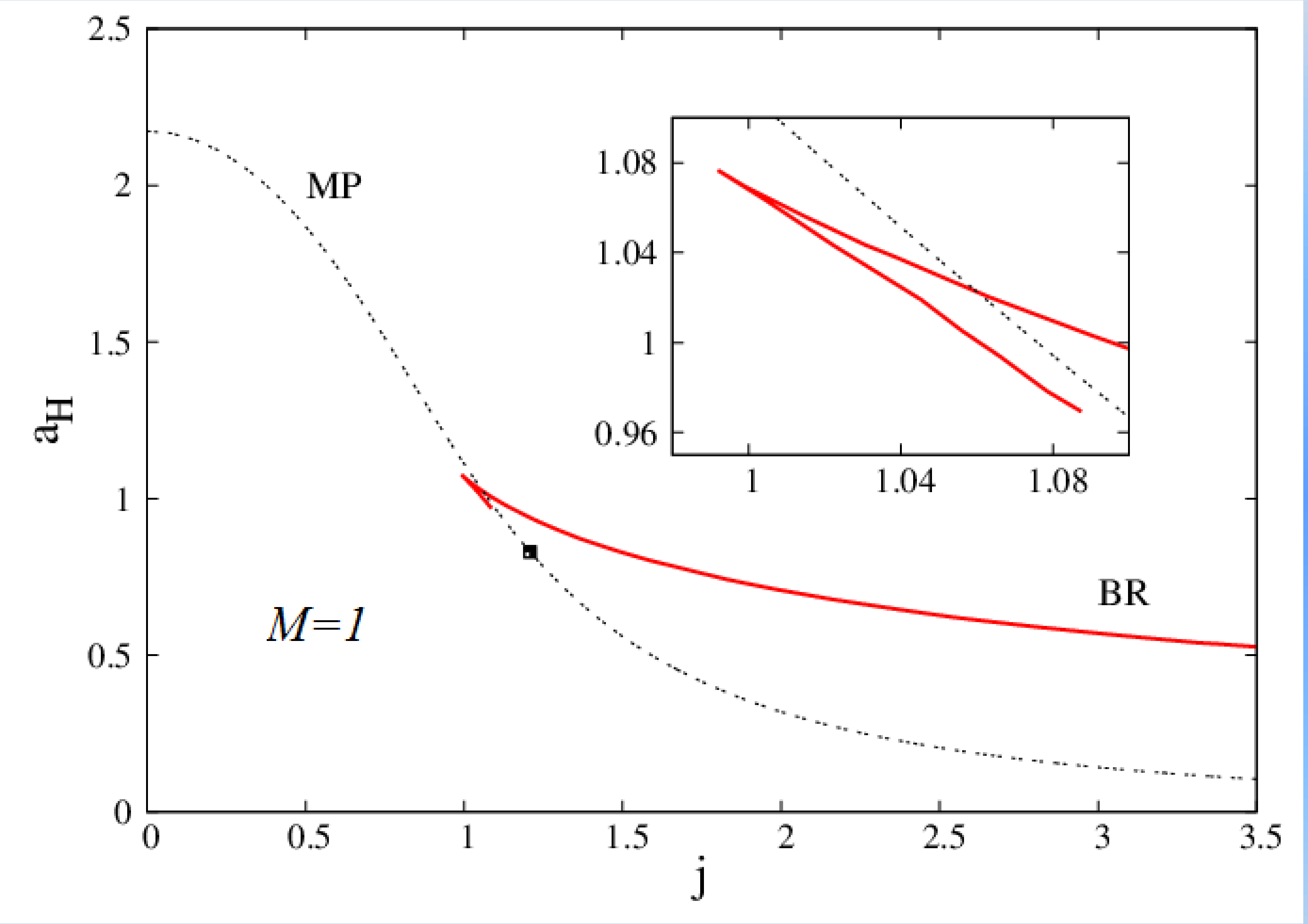
- **fix the scale $r_H=1$**
- **vary $R>r_H$**
- **regular solutions exist for critical values of Ω_H only**

d=6: ~ full set of balanced black rings; *d=7: partial results*



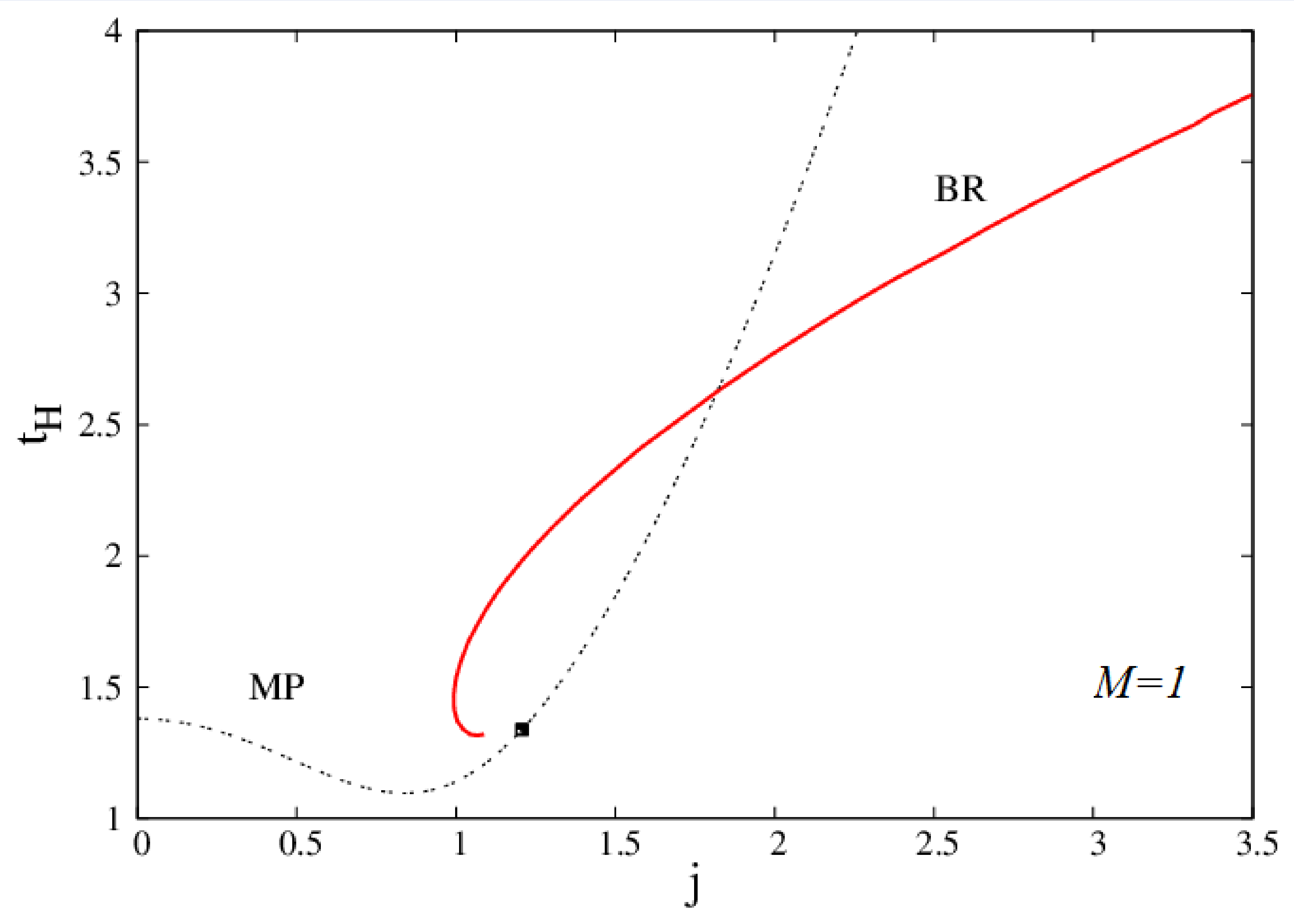
general results: in agreement with expectations from blackfold approach

***some results in six dimensions** (i)*



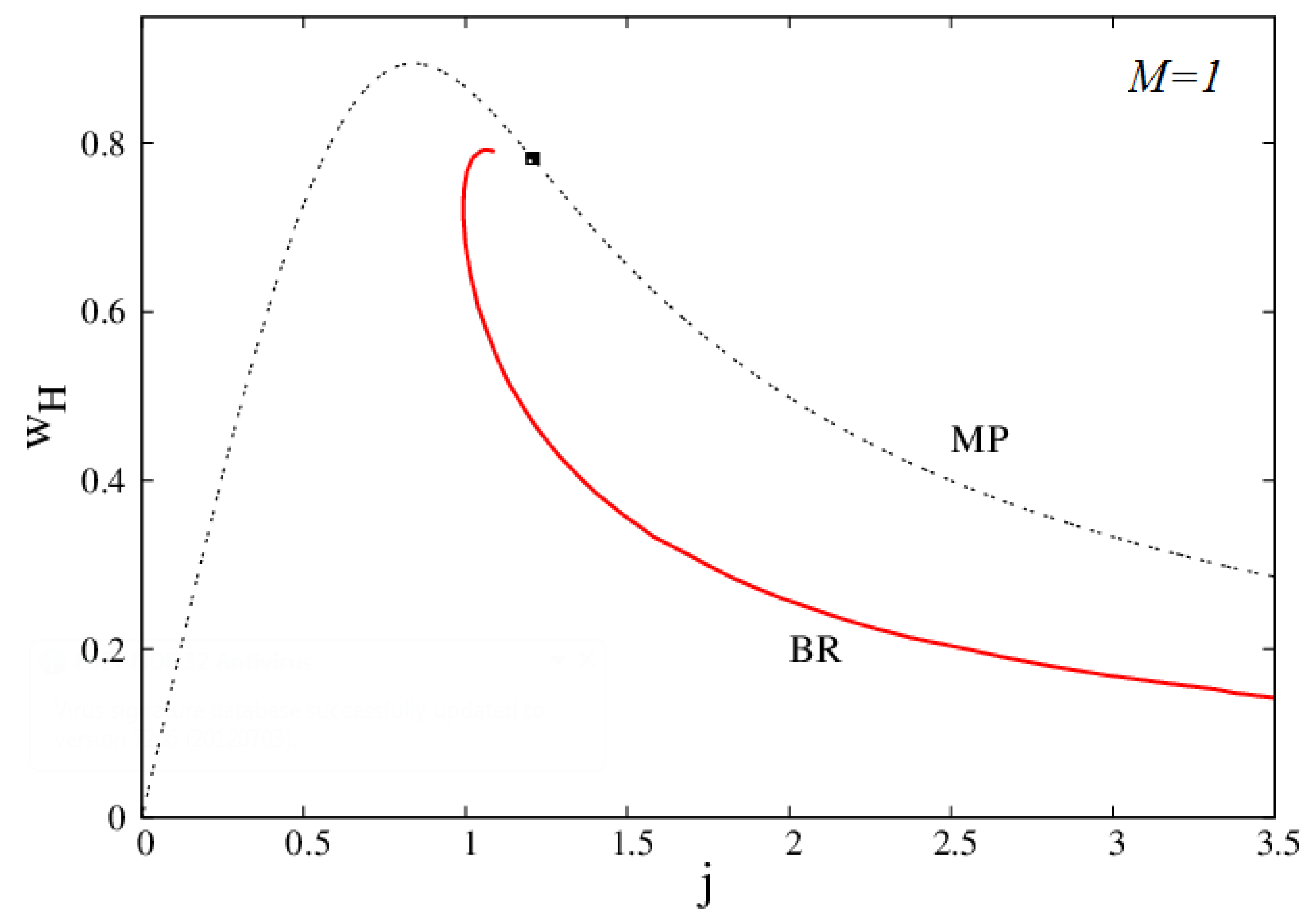
horizon area vs. angular momentum *(scaled quantities)*

***some results in six dimensions** (ii)*



temperature vs. angular momentum *(scaled quantities)*

***some results in six dimensions** (iii)*



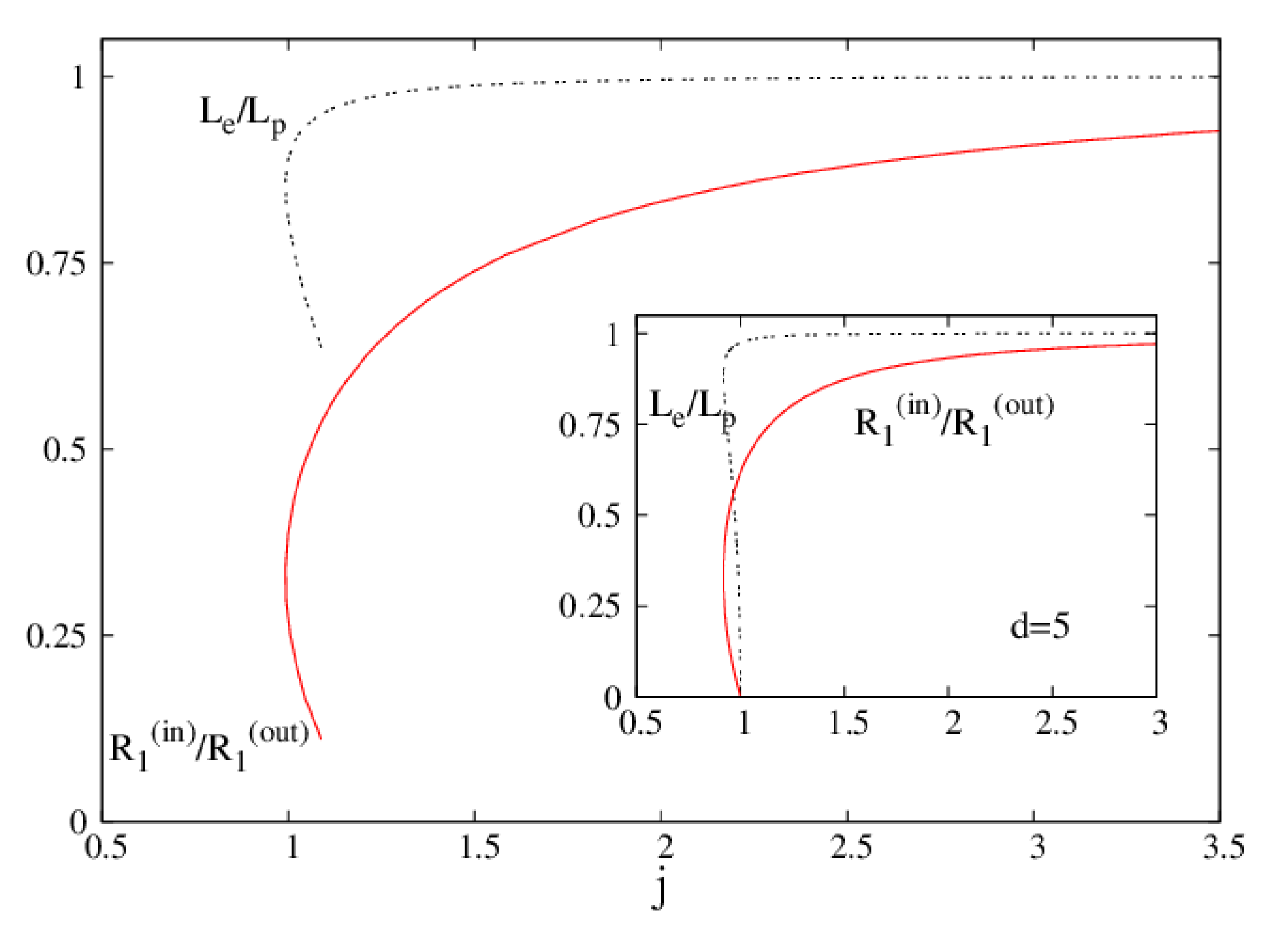
event horizon velocity vs. angular momentum *(scaled quantities)* e.h. metric: $$\boxed{d\sigma^2 = f_1(r_H,\theta) r_H^2 d\theta^2 + f_2(r_H,\theta) d\Omega_{d-4}^2 + f_3(r_H,\theta) d\psi^2}$$

$$L_e = 2\pi \sqrt{f_2(r_H, \pi/4)}$$

the circumference at the equator

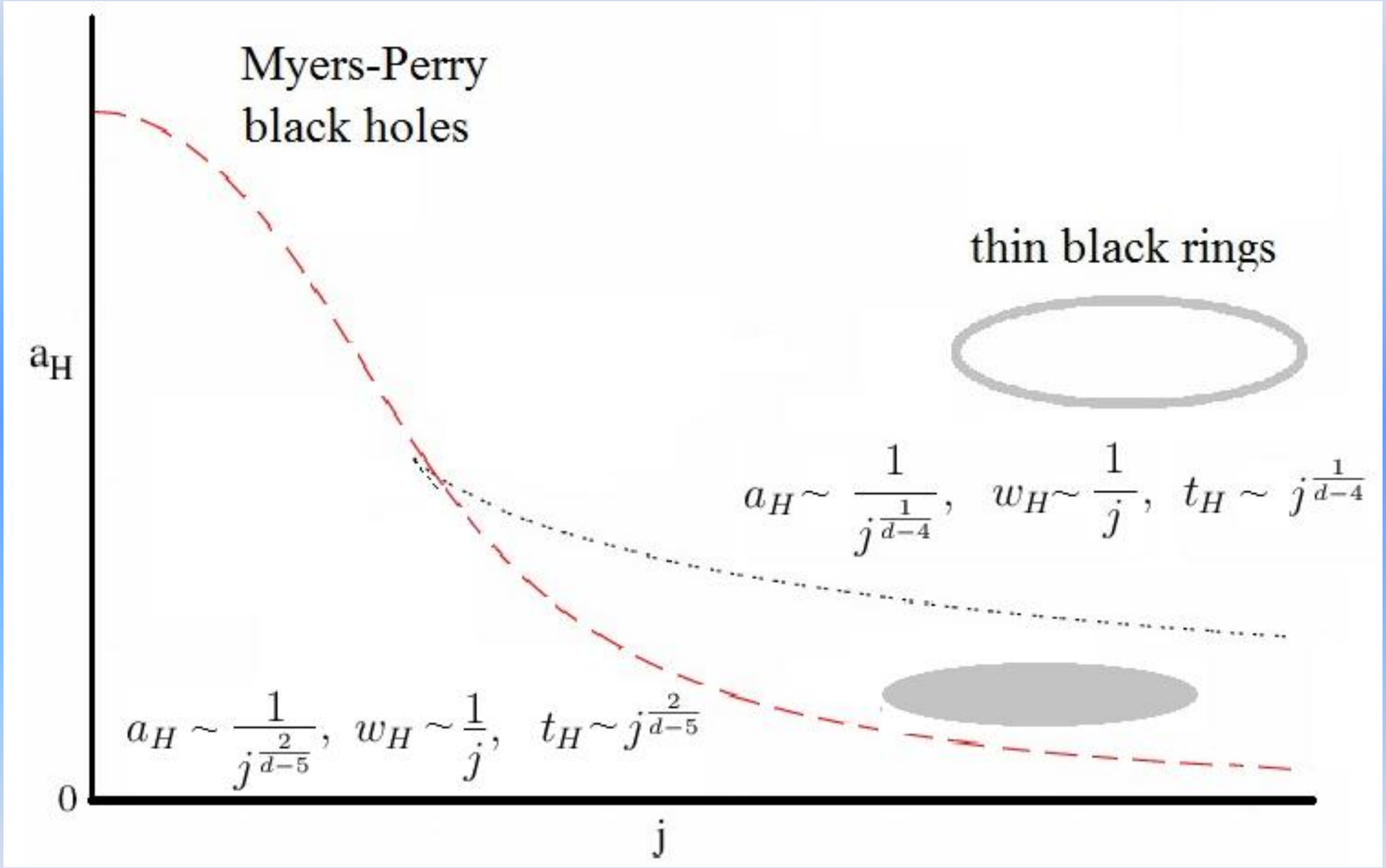
$$L_p = 2\int_0^{\pi/2} d\theta r_H \sqrt{f_1(r_H,\theta)}$$

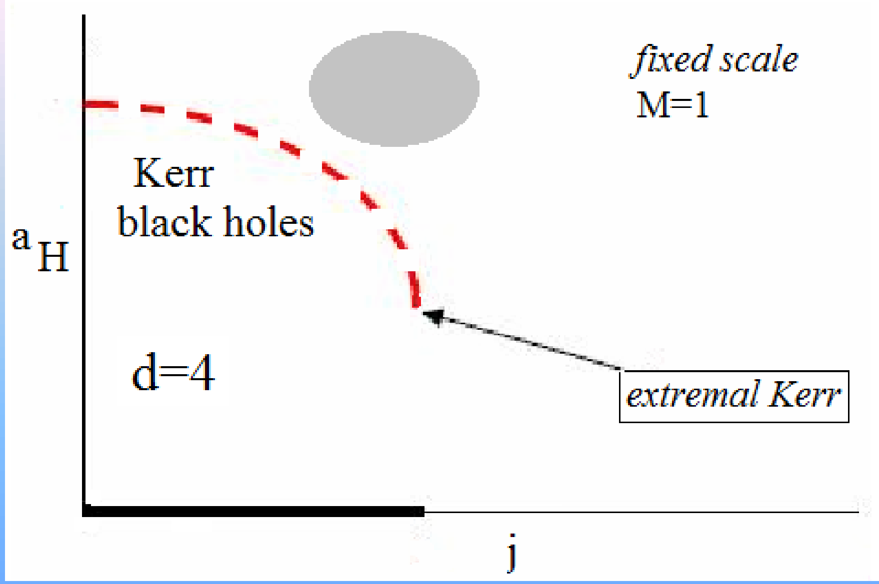
the circumference of S^{d-3} along the poles

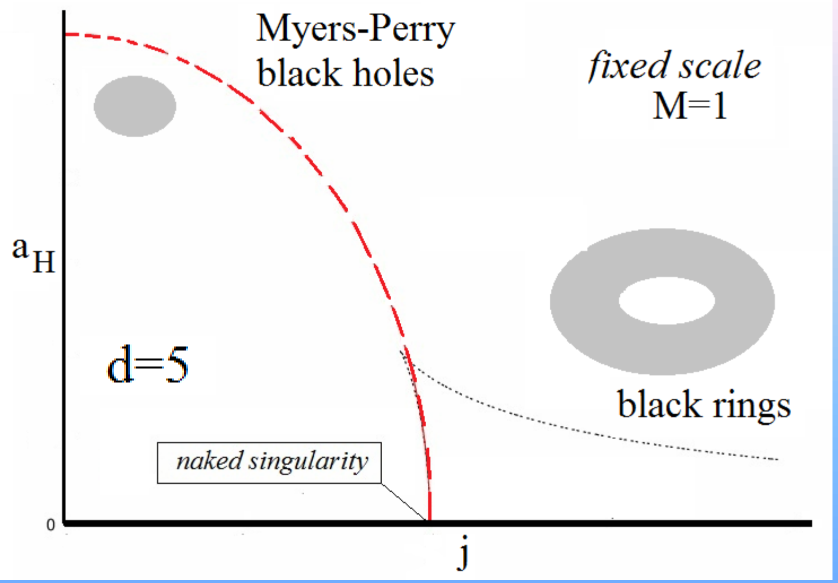


deformation of the horizon

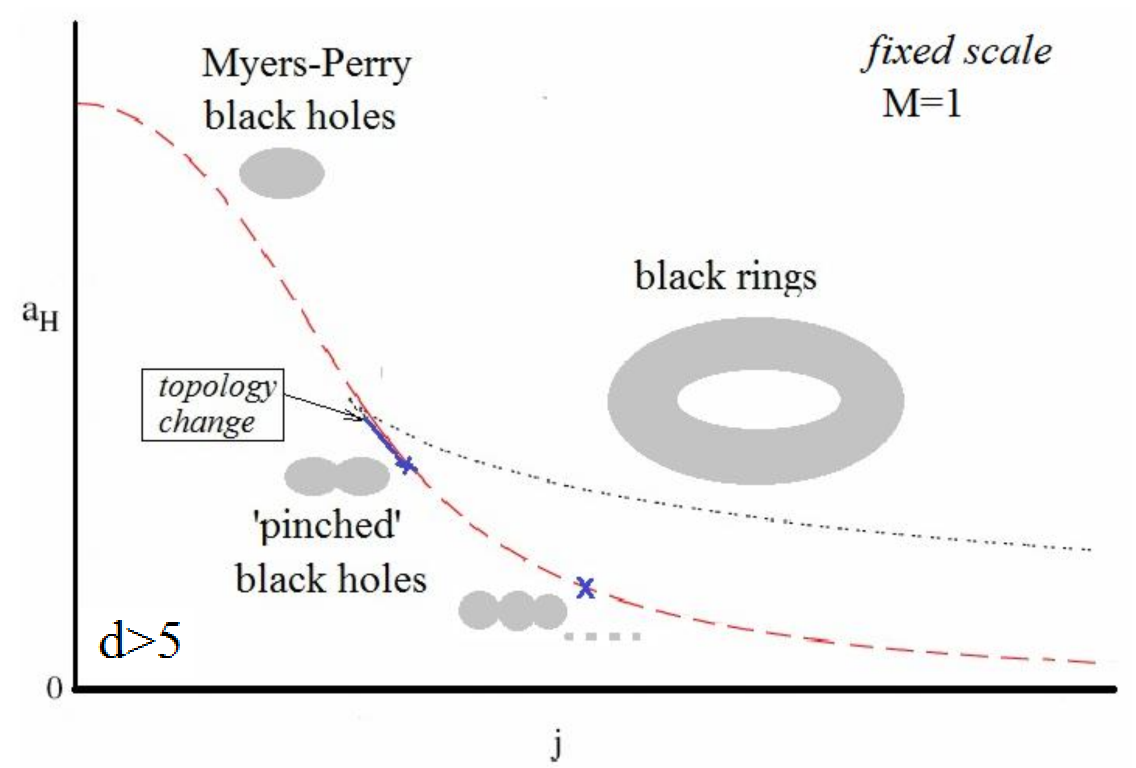
thin black rings: good agreement with analytic results from blackfold approach





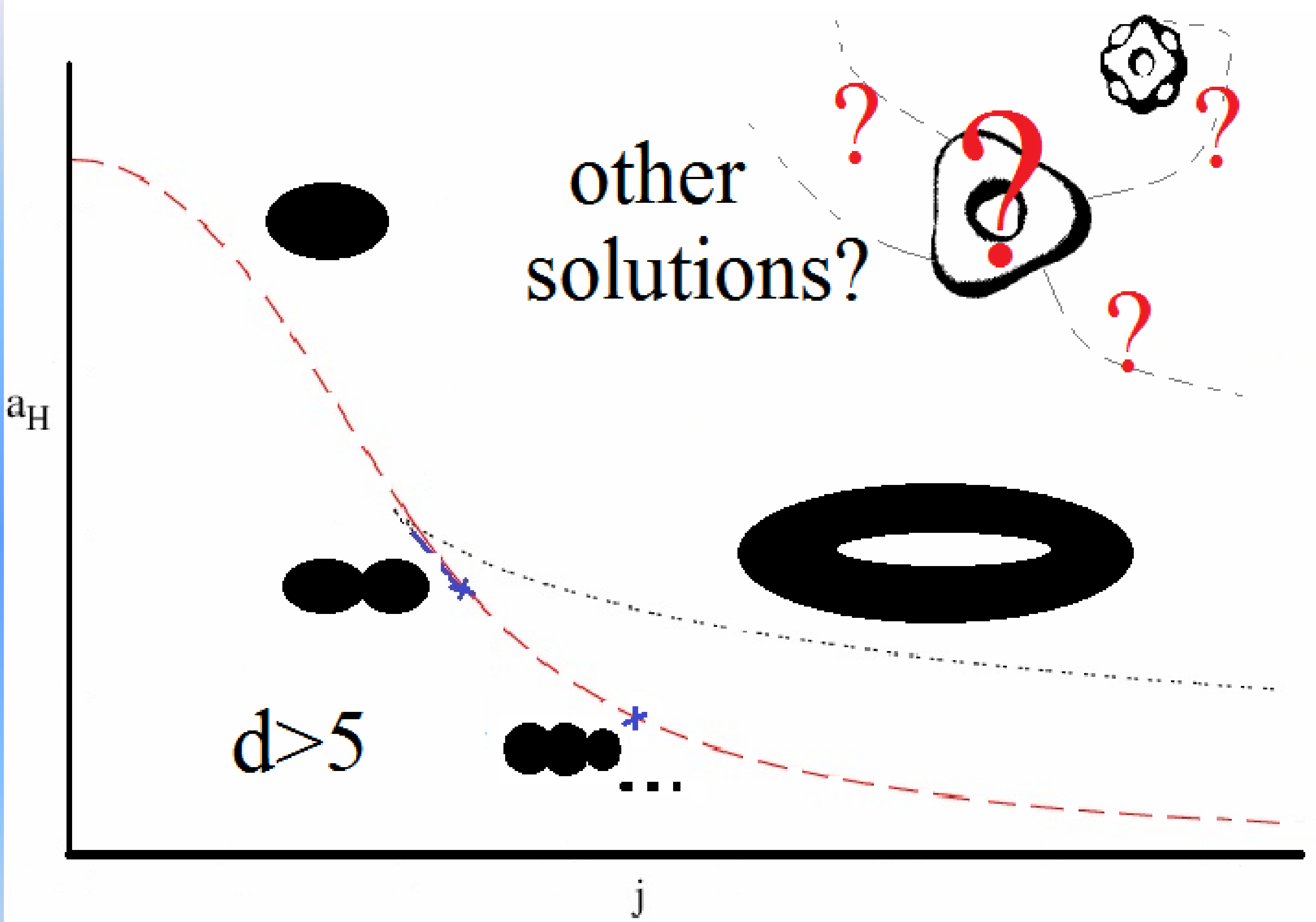


single black objects: emerging picture



(rotation in a single plane)

However, for d>5, the picture is far from being complete

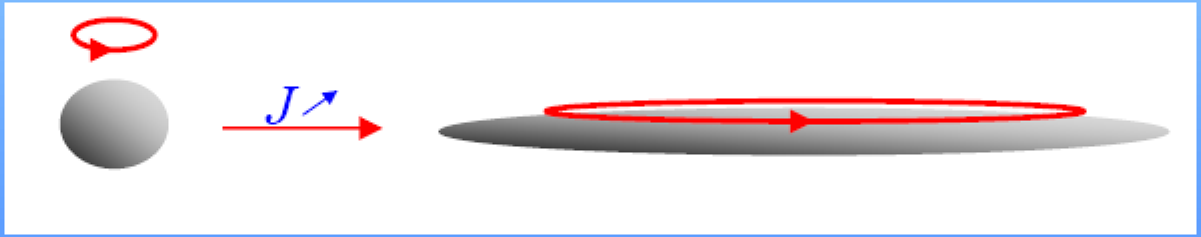


- *many other black holes are likely to exist*
- *blackfold approach*
- *nonperturbative construction?*

Many more exotic BH solutions are likely to exist for d>5

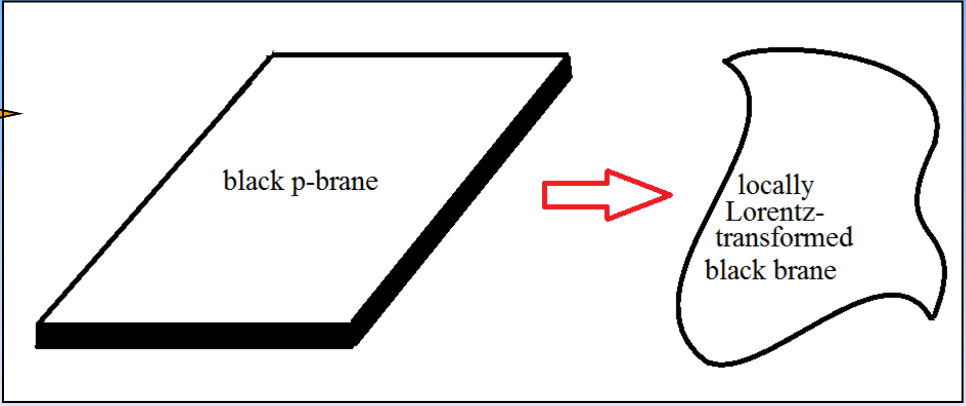
Main obstacle: the d=4,5 formalism+generation techniques do not work for d>5

Myers-Perry (MP) BH - the only one known in closed form
- generalized Kerr: spherical horizon
- d>5: no upper bound on angular momentum



- black brane limit $\Longrightarrow$ GL-type instability

Hints for other BHs:
- blackfolds
- instabilities of MP BHs
- attractors/near horizon geometries
- topology of the horizon: less constrained (*Galloway,Schoen)*



what we know:

dim	*solutions*	*status*
d=4	Kerr: S^2	exact solution
d=5	Myers-Perry: S^3 black ring: $S^2 \times S^1$	exact solution exact solution
d>5	Myers-Perry: S^{d-2} black ring: $S^{d-3} \times S^1$ blackfolds: $\left[S^{d-2-p} \times T^p \atop S^{d-2-m} \times \prod_i S^{m_i} \right.$	exact solution approximation $\left.\right]$ approximation

- is it possible to say something more precise about these solutions?

- new *d>5* black holes?

New black holes with nonspherical horizon topology in more than five dimensions

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(with M. Rodriguez)

static solutions - general framework

vacuum configurations with a symmetry group $R_t \times U(1) \times SO(d-3)$

metric ansatz

$$ds^2 = -f_0(\rho,z)dt^2 + f_1(r,z)(d\rho^2 + dz^2) + f_2(\rho,z)d\psi^2 + f_3(\rho,z)d\Omega_{d-4}^2$$

Rectangular domain suitable for numerics

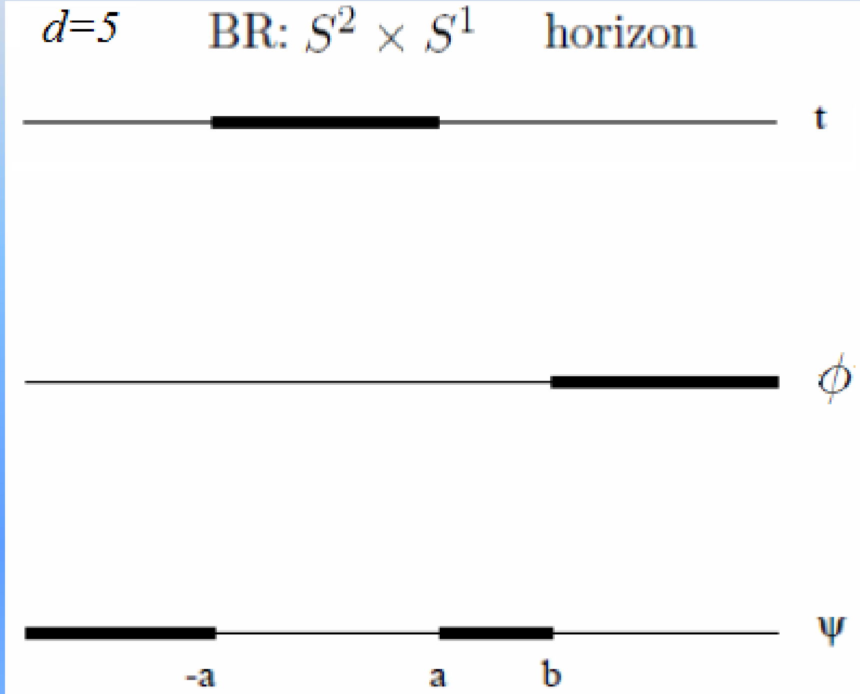
$0 \le \psi \le 2\pi,\ 0 \le \theta \le \pi/2$ $\qquad$ $0 \le \rho < \infty,\ -\infty < z < \infty$

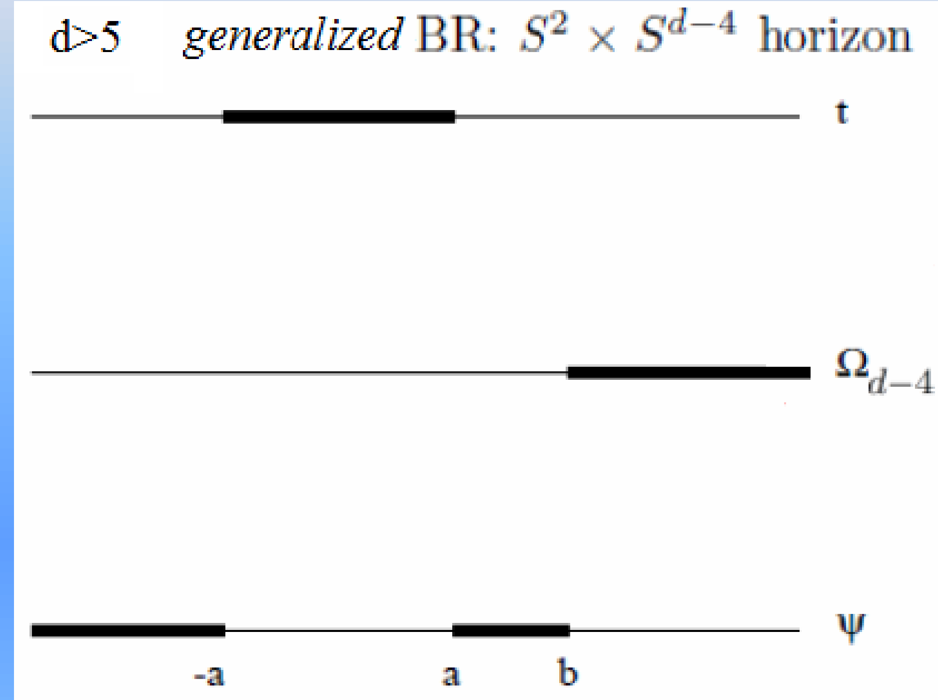
- For d=5 it reduces to standard Weyl coordinates
- Solve (numerically) Einstein equations within this ansatz

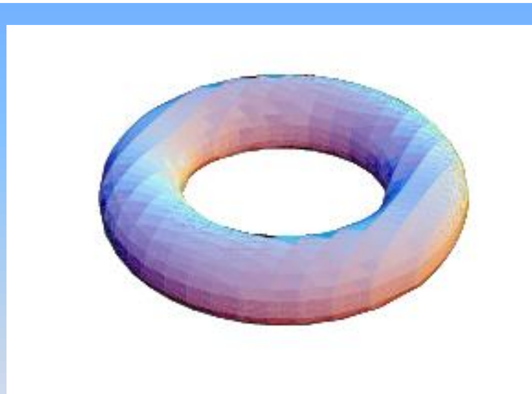
basic element: **generalized rod structure** → fixes the topology of the horizon

(useful tool in d=4,5)

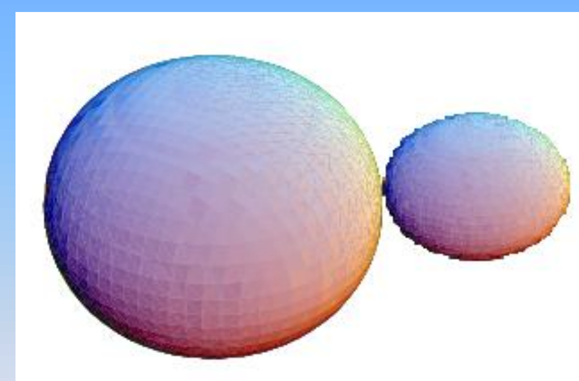
new black holes: ***single black objects*** $S^2 \times S^{d-4}$





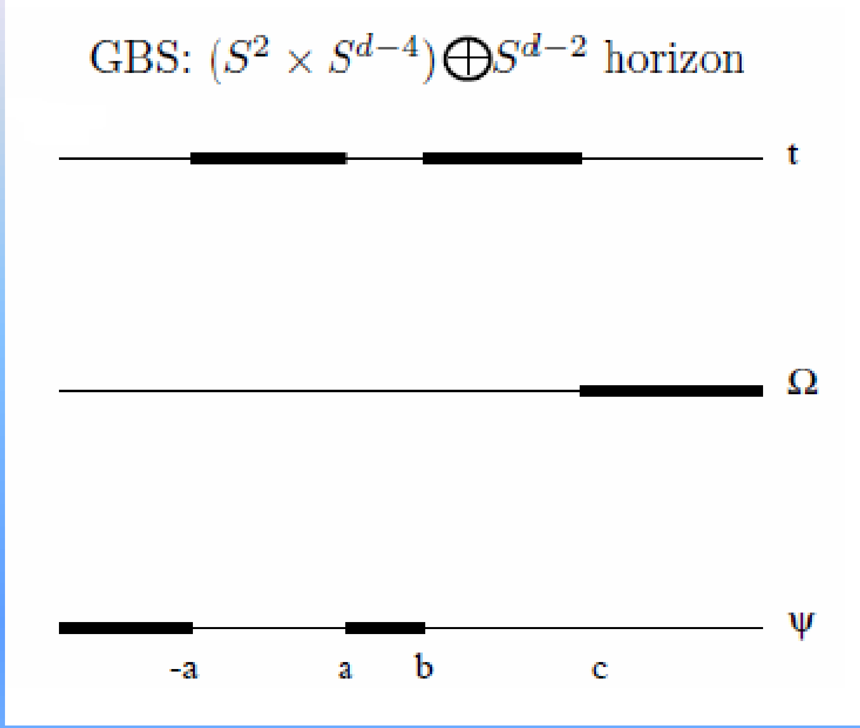


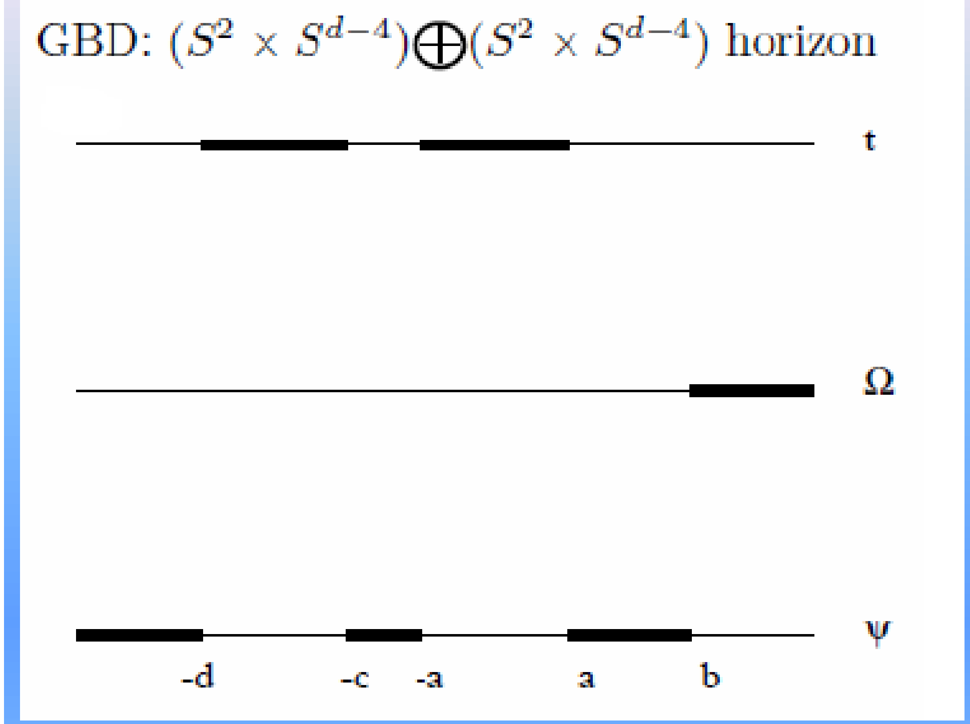
d=5



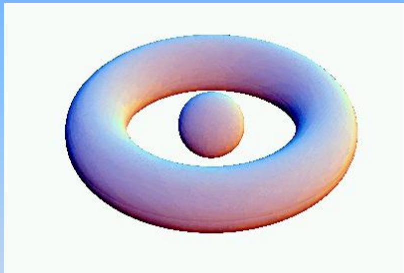
d>5

<u>new black holes</u>: multi-black objects

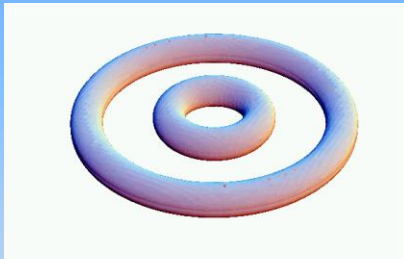




generalized black Saturn



generalized black di-ring

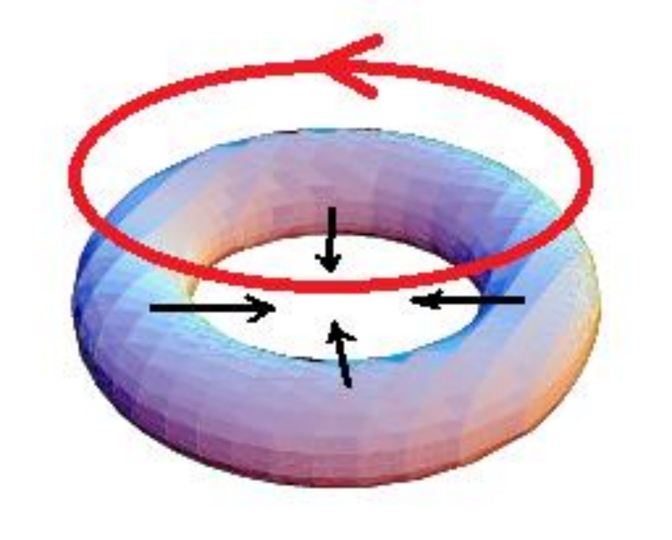


the basic properties of the d=5 known solutions still hold for d>5 generalizations

Rotating solutions

how to balance these solutions?

simplest (only?) solution: add rotation!



No other mechanism is known in this moment for asymptotically flat black holes with non-spherical horizon topology

***d=5 static black rings**: the Gauss-Bonnet term reduces the conical excess, but not enough*

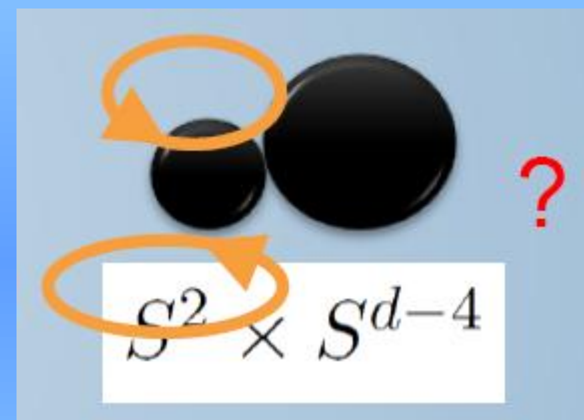
Rotating solutions

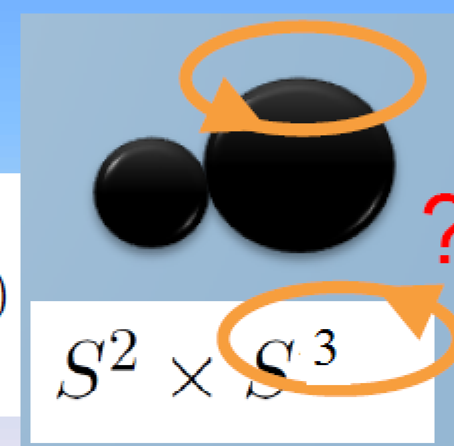
we propose a (slightly) more general ansatz:

(i)

$$ds^2 = -f_0(\rho,z)dt^2 + f_1(\rho,z)\left(d\rho^2 + dz^2\right) + f_2(\rho,z)(d\psi + W(\rho,z)dt)^2 + f_3(\rho,z)d\Omega^2_{d-4}.$$

which solutions can we find?



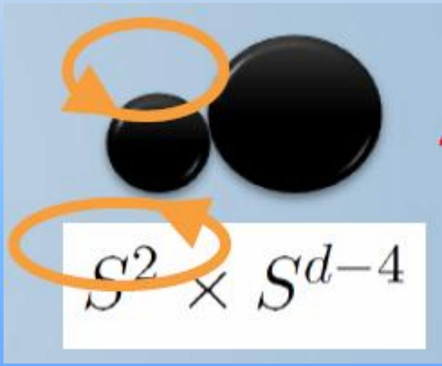


(ii)

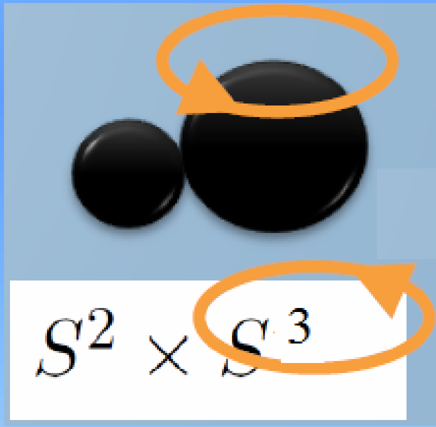
$$\begin{aligned} ds^2 = & -f_0(\rho,z)dt^2 + f_1(\rho,z)(d\rho^2 + dz^2) + f_2(\rho,z)d\psi^2 + f_3(\rho,z)d\theta^2 \\ & + f_4(\rho,z)\left(\sin^2\theta(d\varphi_1 - W(\rho,z)dt)^2 + \cos^2\theta(d\varphi_2 - W(\rho,z)dt)^2\right) \\ & - (f_4(\rho,z) - f_3(\rho,z))\sin^2\theta\cos^2\theta(d\varphi_1 - d\varphi_2)^2, \end{aligned}$$

Rotating solutions

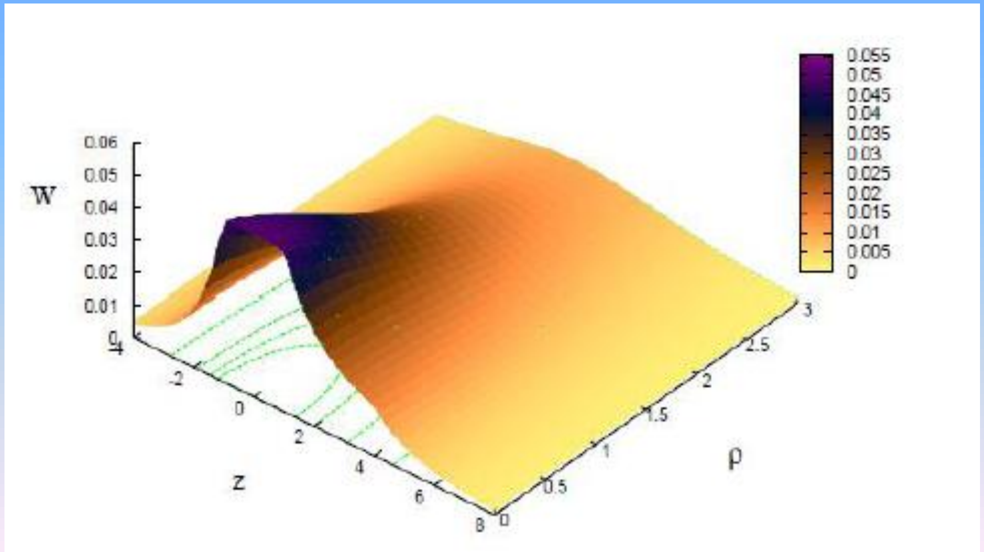
Numerical procedure:
Start from the static solution and increase the event horizon velocity



(i) **d=6:** *we find that there is always a conical singularity;*
we expect that regular configurations are found when
*two spins are present***.**



(ii) **d=7:** *the conical excess decreases with the even horizon*
velocity, suggesting the existence of a critical value
without conical singularities



However, the accuracy is lost before approaching the balanced solution...

Summary:

- higher-*d* black holes: surprisingly rich subject
- we have reported progress in the nonperturbative construction of black holes with nonspherical horizon topology
- **evidence for the existence of d>5 balanced black rings** *(nonperturbative approach)*
- a general framework for the static configurations with an isometry group $R \times U(1) \times SO(d-3)$

  \+ new nonspherical black holes

Thank you very much for your attention!